arXiv:hep-ph/9812253v1  4 Dec 1998

# $b \rightarrow ss\bar{d}$ Decay in Two Higgs Doublet Models

Katri Huitu[a], Cai-Dian Lü[b,d1], Paul Singer[c] and Da-Xin Zhang[a,d]

[a] Helsinki Institute of Physics, P.O.Box 9, FIN-00014 University of Helsinki, Finland

[b] II. Institut für Theoretische Physik, Universität Hamburg, 22761 Hamburg, Germany

[c] Department of Physics, Technion- Israel Institute of Technology, Haifa 32000, Israel

[d] The Abdus Salam International Centre for Theoretical Physics

P.O. Box 586, 34100 Trieste, Italy

**Abstract**

In the Standard Model, the box-diagram mediated decay $b \rightarrow ss\bar{d}$ is predicted to occur with an extremely small branching ratio of below $10^{-11}$, thus providing a safe testing ground for exposing new physics. We study this process in several Two Higgs Doublet Models and explore their parameter space, indicating where this process becomes observable.

---

[1]Alexander von Humboldt research fellow.

# 1

In the study of possible virtual effects in B decays which are due to physics beyond the Standard Model(SM), a crucial task is to subtract reliably the SM contributions. For the best studied rare $b$ decay process, $b \to s + \gamma$, many efforts have been made in the last decade to calculate it as accurately as possible within SM. However, since the signals of new physics in $b$ decays emerge unluckily with branching ratios at the level of $10^{-7}$ or less [1], the commonly focused processes of which $b \to s + \gamma$ is the prototype are very problematic for testing new physics against the SM background. Moreover, other yet unobserved B decays, like various $B \to \tau$ processes, have also been shown to be rather insensitive to a large class of new physics models [2].

In a recent letter [3], we have emphasized an alternative approach to the challenge of identifying virtual effects from new physics in $b$ decays, by the consideration of processes which have negligible strength in the SM. Such processes could thus serve as sensitive probes for new physics, relatively free of SM "pollution". In [3] we focused on the $b \to ss\bar{d}$ transition, which is a box-diagram induced process in the SM with a very small branching ratio of below $10^{-11}$, and we studied possible effects from the minimal supersymmetric standard model (MSSM) and from a supersymmetric model with R-parity violation. We have shown that the existing limits on parameters of MSSM and of R-parity violating interactions allow this process to occur well in excess of the SM rate, thus providing an unusual opportunity for stricter limits or, hopefully, for discovering new physics effects. In the present letter we undertake a study of the occurrence of the $b \to ss\bar{d}$ transition in Two Higgs Doublet Models (THDMs)[4, 5, 6], which are frequently considered as likely candidates for the extension of the SM. We shall study two models in which the charged Higgs exchange is contributing to box diagrams, which we denote as Model I [4] and Model II [5], respectively, as well as a third model allowing a tree level transition mediated by neutral Higgs bosons [6].

We begin with some comments on the calculation of the W-box diagrams in the SM. Due to strong cancellations between the contributions from the top, the charm and the up

quarks in the loops, the leading order SM result for the $b \to ss\bar{d}$ decay rate is

$$\Gamma = \frac{m_b^5}{48(2\pi)^3} \left| \frac{G_F^2}{2\pi^2} m_W^2 V_{tb} V_{ts}^* \right|^2 \left| V_{td} V_{ts}^* f(x) + y V_{cd} V_{cs}^* \, g(x,y) \right|^2, \tag{1}$$

where

$$f(x) = \frac{1 - 11x + 4x^2}{4x(1-x)^2} - \frac{3}{2(1-x)^3} \ln x, \tag{2}$$

$$g(x,y) = -\ln y + \frac{8x - 4x^2 - 1}{4(1-x)^2} \ln x + \frac{4x - 1}{4(1-x)}, \tag{3}$$

with $x = m_W^2/m_t^2$, $y = m_c^2/m_W^2$. The first term in (1) is suppressed by the small Cabibbo-Kobayashi-Maskawa (CKM) matrix elements $V_{td} V_{ts}^*$, while the second term is suppressed by $y = m_c^2/m_W^2$ and contributes about a half of the CKM suppressed term. In this second contribution, we have neglected a kinematics dependent term when we perform the integral over the loop momentum. This amounts to neglecting a small $(m_c^2/m_W^2) \ln(f(p)/m_c)$ contribution wIth $f(p)$ a function depending on the external momenta $p$. We have checked the effect of the neglected dependence numerically and found that it never exceeds 10% of the $m_c^2/m_W^2$ term in the whole kinematic region[2]. Since the involved CKM matrix elements are not well bounded and the relative phase of the two terms is not fixed, we can only determine a range for the branching ratio of $b \to ss\bar{d}$, which turns out to be always below $10^{-11}$ in the SM. Note that QCD corrections may not change the value by orders of magnitudes, if compared with the analogous processes $B^0 \bar{B}^0$ and $K^0 \bar{K}^0$ mixing [7]. All these features combine to single out this process as a very sensitive one to new physics.

The THDMs are the minimal extensions of the SM. With one more Higgs doublet, one has to suppress the tree level flavor changing neutral current (FCNC) interactions due to neutral Higgs bosons to be consistent with the data. The Model I allows only one Higgs doublet to couple to both the up- and the down-type quarks [4]. In the Model II one Higgs doublet is coupled only to up-type quarks while the other doublet is coupled only to down-type quarks [5], and thus the Higgs content in the Model II is the same as in the MSSM. In

---

[2] We note that this dependence does not appear in the calculation of the $K\bar{K}$ mixing, where the external momentum squares are of the order of $m_s^2$ and can be safely neglected compared to $m_c^2$, nor in the case of $B\bar{B}$ mixing, where even the contributions of the order of $m_b^2/m_W^2$ are sub-dominant.
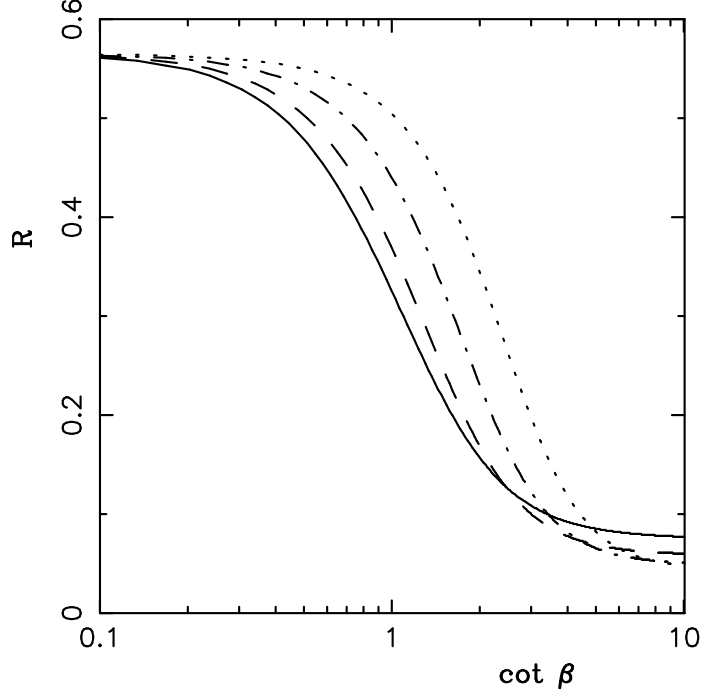
Figure 1: The ratio $R$ of the amplitude induced by the $m_c^2/m_W^2$ term over that by the CKM suppressed one. The solid line, dashed line, dash-dotted line and dotted line correspond to $m_{H^+} = 100, 200, 400, 800$ GeV, respectively.

both the Model I and Model II, discrete symmetries are enforced to forbid the more general couplings and thus the tree level FCNC interactions are absent. In another model, the Model III [6], the tree level FCNC is assumed to be small enough to lie within the experimental bounds.

## 2

The relevant Lagrangian in the Model I and II is

$$\mathcal{L} = \frac{1}{\sqrt{2}}\frac{g_2}{M_W}\left[\cot\beta\ \bar{u}_{i,R}(M_U V)_{ij}d_{j,L} - \xi\bar{u}_{i,L}(M_D V)_{ij}d_{j,R}\right]H^+ + h.c., \tag{4}$$

where $V$ represents the $3 \times 3$ unitary CKM matrix, $M_U$ and $M_D$ denote the diagonalized quark mass matrices, the subscripts $L$ and $R$ denote left-handed and right-handed quarks, and $i, j = 1, 2, 3$ are the generation indices. For model I, $\xi = \cot\beta$; while for Model II, $\xi = -\tan\beta$.

In the charged Higgs mediated box diagrams, the couplings proportional to the up quark masses dominate the contribution to $b \to ss\bar{d}$ decay. In both models we have

$$\Gamma = \frac{m_b^5}{48(2\pi)^3} \left| \frac{G_F^2}{2\pi^2} m_W^2 V_{tb} V_{ts}^* \right|^2 |V_{td} V_{ts}^* \left[ f(x) + A(x,z) \right] + y V_{cd} V_{cs}^* \left[ g(x,y) + B(x,z) \right] |^2, \quad (5)$$

where

$$
\begin{aligned}
A(x,z) &= \frac{\cot^2 \beta}{2} \left( \frac{1-4x}{x(1-z)(1-x)} + \frac{3x}{(1-x)^2(z-x)} \ln x + \frac{z^2-4xz}{x(1-z)^2(z-x)} \ln z \right) \\
&+ \frac{\cot^4 \beta}{4x} \left( \frac{1+z}{(1-z)^2} + \frac{2z}{(1-z)^3} \ln z \right),
\end{aligned}
\quad (6)
$$

$$
\begin{aligned}
B(x,z) &= \cot^2 \beta \left( \frac{4x-z}{2(z-x)(1-z)} \ln z - \frac{3x}{2(1-x)(z-x)} \ln x \right) \\
&- \frac{1}{4} \cot^4 \beta \left( \frac{1}{1-z} + \frac{1}{(1-z)^2} \ln z \right),
\end{aligned}
\quad (7)
$$

with $z = m_{H^+}^2/m_t^2$. The functions $A(x,z)$ and $B(x,z)$ denote the new contributions from the charged Higgs. Because we have dropped in (5) the terms which are proportional to a factor less than $m_b m_s/M_W^2$ at the amplitude level, the limit $\cot \beta \to 0$ corresponds to the SM case. As in the SM, the result depends on the relative CKM phase between the terms with the $m_c^2/m_W^2$ and the CKM suppressed contributions. In Fig. 1 we plot $R$ as the function of $\cot \beta$, where

$$R = \frac{|y V_{cd} V_{cs}^* \left[ g(x,y) + B(x,z) \right]|}{|V_{td} V_{ts}^* \left[ f(x) + A(x,z) \right]|} \quad (8)$$

is the ratio between the amplitudes induced by the $m_c^2/m_W^2$ and by the CKM suppressed terms. It is clear that the SM is the limit with the maximal importance of the $m_c^2/m_W^2$ contribution, while in the limit of large $\cot \beta$ this contribution is negligible.

In the numerical calculation, we use $m_b = 4.5$ GeV, $m_c = 1.5$ GeV, $|V_{ts}| = 0.04$, $|V_{td}| = 0.01$ and $|V_{cd}| = 0.22$. We set the relative phase between the two terms in (5) as zero which corresponds to a maximal constructive interference. In Fig. 2, we show the numerical results for branching ratio of $b \to ss\bar{d}$ decay as a function of $\cot \beta$ for different charged Higgs masses. We have taken $m_{H^+}$ as 100–800 GeV while $\cot \beta$ ranging from 0.1 to 5. Some semi-quantitative considerations suggest a small $\cot \beta$ $(< 2)$,[3] in which region the

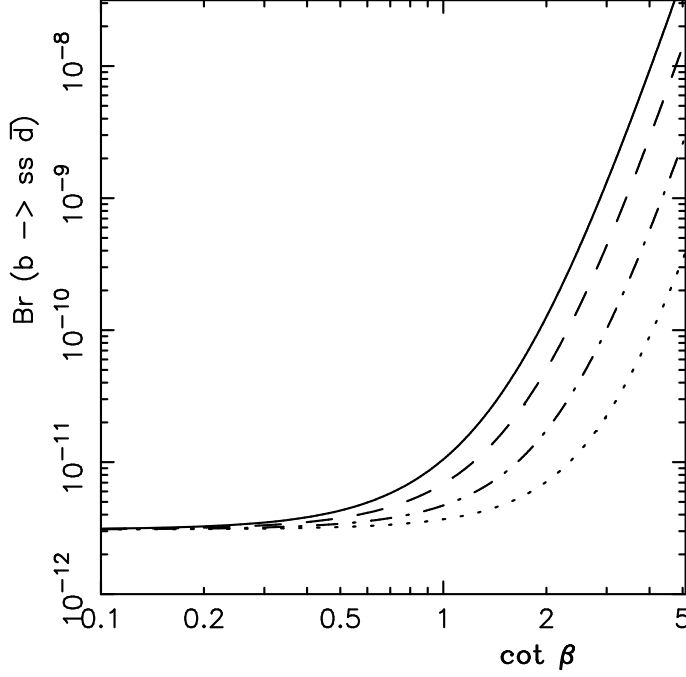---

[3] We refer the reader to [8] for more detailed discussions.

Figure 2: The branching ratio of $b \to ss\bar{d}$ as a function of $\cot\beta$. The solid line, dashed line, dash-dotted line and dotted line correspond to $m_{H^+} = 100, 200, 400, 800$ GeV, respectively. Regions above the lines are excluded.

decay branching ratio for $b \to ss\bar{d}$ decay is unobservable. However, phenomenologically a $\cot\beta$ as large as 5 is still consistent with the low energy data [9] and the direct search of the Higgs boson in top decays. In this parameter space of a large $\cot\beta$ the charged Higgs box diagrams are sizable. We note that in the MSSM $\cot\beta < 1$ is preferred in model building, corresponding to an unobservable contribution from the charged Higgs box diagrams in this model [3].

As can be seen in Fig. 2, the branching ratio in the Model I and II can be of the order $10^{-8}$ at large $\cot\beta$ region. The searching of the decay $b \to ss\bar{d}$ and its hadronic channels $B^\pm \to K^\pm K^\pm X$ in B experiments will further constrain the parameters in the THDMs.

## 3

In the THDM III, there exists the Yukawa interaction [6]

$$\mathcal{L} = \xi_{ij}\bar{Q}_{i,L}\phi_2 D_{j,R} \; + \; (\text{the up quark sector}) \; + \; h.c..$$ (9)

6

Here, the scalar Higgs doublet $\phi_2$ mediates the FCNC transitions $d_i \leftrightarrow d_j$ at the tree level, if the coupling $\xi_{ij}$ is nonzero. As discussed in the literature, FCNC effects induced by the loop diagrams are always negligible compared to the tree level ones in the Model III [10], hence the box diagrams with charged Higgs can be safely dropped in the present decay. We will neglect the unimportant QCD correction since no GIM-like cancellation happens in the process.

The couplings in (9) are constrained strongly from the neutral meson mixing by the requirement that the FCNC contribution to the mixing from the interactions in (9) does not exceed its experimental value. In the $K\bar{K}$ and $B_s\bar{B}_s$ mixing, the dominant contributions are proportional to $\xi_{sd}^2$ and $\xi_{sb}^2$, respectively [10]

$$\Delta M_F = 2\xi_{sq}^2 \left( \frac{M_S^F}{m_h^2} + \frac{M_P^F}{m_A^2} \right), \tag{10}$$

with $F = K, B_S$, and $q = d, b$, respectively, and

$$
\begin{aligned}
M_S^F &= \frac{1}{6}\left( f_F^2 M_F + \frac{f_F^2 M_F^3}{(m_s + m_q)^2} \right), \\
M_P^F &= \frac{1}{6}\left( f_F^2 M_F + \frac{11 f_F^2 M_F^3}{(m_s + m_q)^2} \right).
\end{aligned}
\tag{11}
$$

Here $m_h$ and $m_A$ are the masses of the neutral scalar and pseudoscalar Higgs bosons. Note that we have taken $\xi_{ij} = \xi_{ji}$ while it is not difficult to generalize to the case without this assumption. In numerical calculations, we use $f_K = 160$ MeV, $f_{B_s} = 200$ MeV, $\Delta M_K = 3.491 \times 10^{-15}$ GeV [11]. Taking $m_A = m_h \equiv m_H$, we obtain the bound

$$\frac{\xi_{sd}}{m_H} < 8.3 \times 10^{-8} \text{GeV}^{-1} \tag{12}$$

from $\Delta M_K$. The experimental lower limit from $B_s - \bar{B}_s$ mixing

$$\Delta M_{B_s} > 5.2 \times 10^{-12} \text{GeV} \quad [12], \tag{13}$$

gives no constraint on $\xi_{sb}/m_H$, since the contribution in the SM itself exceeds this number. Furthermore, free from any assumption made for the FCNC Higgs couplings in the lepton sector, the bounds from $B_s \to l_i l_j$ and $B \to K l_i l_j$ ($l_i, l_j = e, \mu$ or $\tau$) do not exclude even $\xi_{sb}/m_H$ as large as $10^{-1}$ [13].
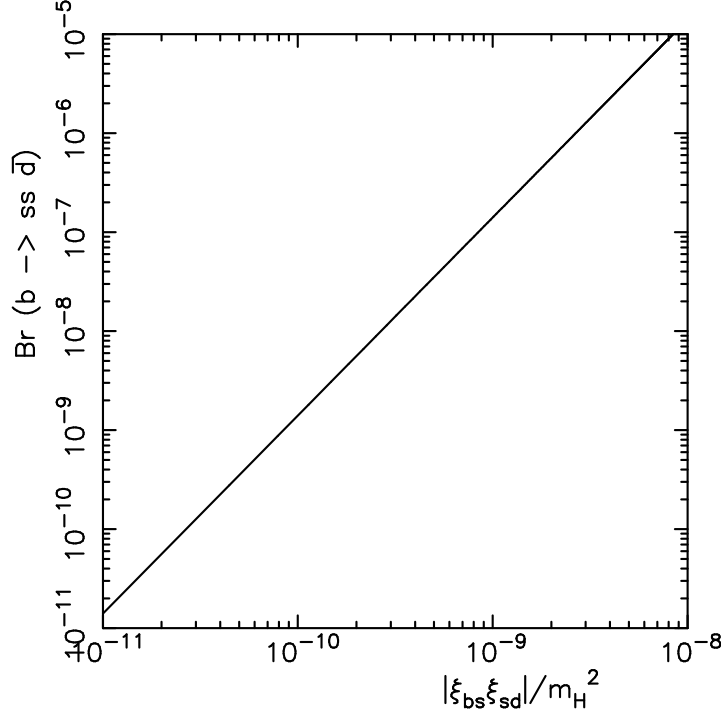
Figure 3: The branching ratio of $b \to ss\bar{d}$ in THDM III as a function of $|\xi_{bs}\xi_{sd}|/m_H^2$.

In the presence of the interactions (9), $b \to ss\bar{d}$ can be induced by a tree diagram exchanging the neutral Higgs bosons $h$ (scalar) and $A$ (pseudo-scalar), with the amplitude

$$\mathcal{A} = \frac{i}{2} \xi_{sb}\xi_{sd} \left( \frac{1}{m_h^2}(\bar{s}b)(\bar{s}d) - \frac{1}{m_A^2}(\bar{s}\gamma_5 b)(\bar{s}\gamma_5 d) \right). \tag{14}$$

The decay rate is thus

$$\Gamma = \frac{m_b^5}{3072(2\pi)^3}|\xi_{sb}\xi_{sd}|^2 \left\{ 11 \left( \frac{1}{m_h^4} + \frac{1}{m_A^4} \right) + \frac{2}{m_h^2 m_A^2} \right\}. \tag{15}$$

The numerical results are shown in Fig.3 as a function of $|\xi_{sb}\xi_{sd}|/m_H^2$.

From Fig. 3, it can be seen that the decay $b \to ss\bar{d}$ is observable in the Model III, if $|\xi_{sb}\xi_{sd}|/m_H^2 > 10^{-10}$ GeV$^{-2}$, which corresponds to a branching ratio around $10^{-9}$. This requires at least $|\xi_{sb}/m_H| > 10^{-3}$ GeV$^{-1}$. Corresponding to this number, the neutral Higgs contribution to $\Delta M_{B_s}$ is $10^6$ times larger than its present lower limit. To our knowledge, such a large $\Delta M_{B_s}$ is difficult to exclude.

**4**

In summary, we have presented a study of the $b \to ss\bar{d}$ process in several THDMs. Firstly, we confirmed that the charged Higgs box contribution in MSSM is indeed negligible [3], while on the other hand in Models I [4] and II [5] this contribution can induce observable effects at the $10^{-8}$ level for the branching ratio. A large $B_s\bar{B}_s$ mixing, which is at least $10^6$ times larger than its present lower limit, is required in Model 3 [6] for this process to be observable. Combining the above results with those of Ref [3], we reemphasize that the search for the processes we recommended [3], like $B^- \to K^-K^-\pi^+$, will serve immediately to get better limits on the parameters of R-parity violating theories; at the later stage, when lower experimental limits are obtainable, it would constitute a direct check of the various Non-Standard Model theories investigated here and in Ref [3].

# References

[1] For reviews, see A. Masiero and L. Silvestrini, hep-ph/9709244, in proceedings of the 2nd International Conference on B Physics and CP Violation (BCONF 97), Honolulu, HI, Mar 1997, eds. T.E. Browder, F.A. Harris, S. Pakvasa, World Scientific (1998); J.L. Hewett, hep-ph/9610501, in the proceedings of the 28th International Conference on High Energy Physics, Warsaw, Poland, July 1996, eds. Z. Ajduk, A.K. Wroblewski, World Scientific (1997).

[2] D.Guetta and E.Nardi, Phys. Rev. **D58** (1998) 012001; see here for an extensive list of previous references.

[3] K. Huitu, C.-D, Lü, P. Singer, D.-X. Zhang, hep-ph/9809566, preprint DESY 98-037, HIP-1998-07/TH, TECHNION-PH-11, to appear in Phys. Rev. Lett.

[4] H.E. Haber, G.L. Kane and T. Sterling, Nucl. Phys. **B161** (1979) 493.

[5] S.L. Glashow and S. Weinberg, Phys. Rev. **D15** (1977) 1958.

[6] T. P. Cheng and M. Sher, Phys. Rev. **D35** (1987) 3484; M. Sher and Y. Yuan, Phys. Rev. **D44** (1991) 1461.

[7] A. J. Buras, M. Jamin and P. H. Weisz, Nucl. Phys. **B347** (1990) 491.

[8] K. Kiers and A. Soni, Phys. Rev. **D56** (1997) 5786.

[9] V. Barger, J.L. Hewett and R.J.N. Phillips, Phys. Rev. **D41** (1990) 3421.

[10] D. Atwood, L. Reina and A. Soni, Phys. Rev. **D55** (1997) 3156.

[11] Particle Data Group, Phys. Rev. **D54** (1996) 1.

[12] R. Barate et al., ALEPH Collaboration, preprint CERN-PPE-97-157.

[13] See M. Sher, hep-ph/9809590 and references therein.